Benha University | **Antennas &Wave Propagation** | **Electrical Eng. Dept.**
Faculty of Engineering | | **4th year communication**
Shoubra | | **2015-2016**

Sheet (2)... (Solution)

1. A hypothetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field ($\mathbf{E_\theta}$) is measured to be 5 V/m. Find the

(a) Power density (W_{rad})

(b) Power radiated (P_{rad})

$$(a)\quad \underline{W}_{rad} = \tfrac{1}{2}[\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta}\hat{a}_r = \frac{5^2 \hat{a}_r}{2(120\pi)} = 0.03315\, \hat{a}_r \text{ Watts/m}^2$$

$$(b)\quad P_{rad} = \oint_S W_{rad}\, dS = \int_0^{2\pi}\int_0^{\pi} (0.03315)(r^2 \sin\theta\, d\theta\, d\phi)$$

$$= \int_0^{2\pi}\int_0^{\pi} (0.03315)(100)^2 \sin\theta\, d\theta\, d\phi$$

$$= 2\pi\,(0.03315)(100)^2 \int_0^{\pi} \sin\theta\, d\theta = 2\pi(0.03315)(100)^2 \cdot 2$$

$$= 4165.75 \text{ watts}$$

2. Estimate the directivity of an antenna with $\mathbf{\Theta_{HP} = 2^o}$ and $\mathbf{\Phi_{HP} = 1^o}$.

$$D\ approximate = \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{2*1} = 20627.$$

3. Find the number of square degrees in the solid angle Ω on a spherical surface that is between ($\mathbf{\theta=20^o}$ and $\mathbf{\theta=40^o}$), and ($\mathbf{\varphi=30^o}$ and $\mathbf{\varphi=70^o}$).

$$\Omega = \int_{30}^{70} d\phi \int_{20}^{40} \sin\theta d\theta . = (70\text{-}30)*(\frac{180}{\pi}) * (-Cos\,\theta)_{20}^{40} = 398.17\ deg^2.$$

4. The radiation intensity of antenna is given by $\mathbf{U=B_oCos\theta}$. U exists only in the upper hemisphere, Find

a. The exact directivity.
b. The approximate directivity.
c. The decibel difference.

U =U_n =P_n =Cosθ.

$$\textit{(a) Dexact} = \frac{4\Pi}{\int_0^{2\Pi}\int_0^{\frac{\Pi}{2}} Cos\,\theta\ Sin\theta d\theta d\phi} = \frac{4\Pi}{(2\Pi)(\frac{-Cos^2\theta}{2})_0^{\frac{\Pi}{2}}} = 4.$$

Dr. Gehan Sami
M.M. Elsherbini

(b)D approximate $= \dfrac{4\Pi}{\theta_{HP}\phi_{HP}} .= \dfrac{4\Pi}{\theta_{HP}\phi_{HP}} = \dfrac{4\Pi}{(\theta_{HP})^2}\Big|_{sr} = \dfrac{41253}{(\theta_{HP})^2}\Big|_{\deg 2}.$

We calculate θ_{max} ➔ $(Cos\theta_{max} = 1)$ ➔ *at* $\theta_{max} = 0^o$,

We calculate θ_h ➔ $(Cos\theta_h = \frac{1}{2})$ ➔ $\theta_h = 60^o$

$\theta_{HP} = 2*|\theta_{max} - \theta_h| = 2*|0^o - 60^o| = 120^o$

so : Dapprox. $= \dfrac{41253}{(\theta_{HP})^2}\Big|_{\deg 2}. = = \dfrac{41253}{(120)^2} = 2.86.$

(c) *Decibel difference = 10 log* $\dfrac{4}{2.86}$ *= 1.46db.*

5. An antenna has a field pattern given by $\mathbf{E(\theta) = \cos^2\theta}$, For $0 \le \theta^o \le 90^o$. Find the beam area of this pattern.

1- Exact... $\Omega_A = \int_0^{2\Pi}\int_0^{\Pi/2} Cos^4\theta \sin\theta d\theta d\phi = -2\Pi*(\frac{1}{5}Cos^5\theta)_0^{\Pi/2} = 1.26Sr.$

2- Approximate... $\Omega_A = \theta_{HP}\phi_{HP} .= (\theta_{HP})^2$

<u>To obtain θ_{HP}</u>... we must firstly calculate $\theta\max$, θ_h

<u>To Obtain θ_{max}</u>... the angle of which $P_n = E^2_n$ *maximum.*

It occurs when $cos^2\theta = 1$ *... at* $\theta_{max} = 0^o$.

<u>To obtain θ_h</u>... the angle of which $Pn = \frac{1}{2}$. *(Or* $E_n = \frac{1}{\sqrt{2}}$*).*

It occurs when $cos^2\theta_h = \dfrac{1}{\sqrt{2}}$.

So: $\theta_h = 32.76^o \cong 33^o$.

<u>Now</u>: $\theta_{HP} = 2*|\theta\max - \theta_h| = 2*|0-33| = 66^o$.

So: $\Omega_A = (\theta_{HP})^2 = (66)^2 = 4356\ deg^2 = 4356*(\Pi/180)^2 = 1.33Sr.$

6. The normalized field pattern of an antenna is given by $\mathbf{E(\theta) = \sin\theta\sin\varphi}$. $\mathbf{E_n}$ has a value only for $0 \le \theta \le \Pi$ & $0 \le \varphi \le \Pi$, and zero elsewhere , Find

a. The exact directivity.
b. The approximate directivity.
c. The decibel difference.

Dr. Gehan Sami
M.M. Elsherbini

(a)* *Dexact $= \dfrac{4\Pi}{\int_0^{\Pi}\int_0^{\Pi} Sin^3\theta Sin^2\phi d\theta d\phi} = \dfrac{4\Pi}{\int_0^{\Pi} \sin\theta * (1 - Cos^2\theta) d\theta \int_0^{\Pi} \frac{(1 - Cos 2\Phi)}{2} d\phi} =$

$$\frac{4\Pi}{(\frac{\phi}{2} - \frac{Sin 2\phi}{2})_0^{\Pi} \int_0^{\Pi} (Sin\theta d\theta - Sin\theta Cos^2\theta d\theta)} = \frac{4\Pi}{(\frac{\Pi}{2})[(-Cos\theta)_0^{\Pi} + (\frac{Cos^3\theta}{3})_0^{\Pi}]} =$$

$$\frac{4\Pi}{(\frac{\Pi}{2})(\frac{4}{3})} = 6.$$

(b).. *D approximate* $= \dfrac{4\Pi}{\theta_{HP}\phi_{HP}}$.

We calculate θ_{max} ➔ $(Sin\theta_{max} = 1..(max))$ ➔ *at* $\theta_{max} = 90^o$,
We calculate Φ_{max} ➔ $(Sin\Phi_{max} = 1..(max))$ ➔ *at* $\Phi_{max} = 90^o$,

We calculate θ_h ➔ $(Sin\theta_h = \frac{1}{\sqrt{2}})$ ➔ $\theta_h = 45^o$

We calculate Φ_h ➔ $(Sin\Phi_h = \frac{1}{\sqrt{2}})$ ➔ $\Phi_{h} = 45^o$

So: $\theta_{HP} = 2*|90-45^o| = 90^o = \frac{\Pi}{2} (rad)$

By the same way

We calculate $\Phi_{HP} = 2*|90^o - 45^o| = 90^o = \frac{\Pi}{2} (rad)$

So: D approximate $= \dfrac{4\Pi}{\theta_{HP}\phi_{HP}} . = \dfrac{4\Pi}{(\frac{\Pi}{2})(\frac{\Pi}{2})} = 5.1.$

(C) *Decibel difference* $= 10 \log \frac{6}{5.1} = 0.7db.$

7. The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of
$\mathbf{U = B_o cos^3\theta}$ (watts/unit solid angle) $(0 \le \theta \le \pi/2,\ 0 \le \varphi \le 2\pi)$
Find the
(a) Maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.
(b) Exact and approximate beam solid angle Ω_A.

Dr. Gehan Sami
M.M. Elsherbini

(c) Directivity, exact and approximate, of the antenna (dimensionless and in dB).

(d) Gain, exact and approximate, of the antenna (dimensionless and in dB).

$U = B_o \cos^3\theta$

(a) $P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \sin\theta \, d\theta \, d\phi = B_o \int_0^{2\pi}\int_0^{\pi/2} \cos^3\theta \sin\theta \, d\theta \, d\phi$

$= 2\pi B_o \int_0^{\pi/2} \cos^3\theta \sin\theta \, d\theta$

$P_{rad} = 2\pi B_o \left(-\frac{\cos^4\theta}{4}\right)\Big|_0^{\pi/2} = \frac{\pi}{2} B_o = 10 \Rightarrow B_o = \frac{20}{\pi} = 6.3662$

$U = 6.3662 \cos^3\theta$

$W = \frac{U}{r^2} = \frac{6.3662}{r^2}\cos^3\theta = \frac{6.3662}{(10^3)^2}\cdot\cos^3\theta = 6.3662\times10^{-6}\cos^3\theta$

$W|_{max} = 6.3662\times10^{-6}\cdot\cos^3\theta\Big|_{max} = 6.3662\times10^{-6}$ Watts/m²

(b) $D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(6.3662)}{10} = 8 = 9\,dB$

(c) $G_o = e_t D_o = 8 = 9\,dB$

8. Calculate the $D_{approx.}$ from the HPBW of a unidirectional antenna if the power pattern is given by :

$$E(\theta,\phi) = 30 \cos^2\theta \sin^{3/2}\Phi$$

$0 \le \theta \le \Pi \qquad 0 \le \Phi \le \Pi$ and zero otherwise.

Then repeat by calculating D_{exact} for the previous pattern. Finally calculate the db difference between the exact and approximate records.

$$\textit{Dexact} = \frac{4\Pi}{\int_0^{\Pi}\int_0^{\Pi} E_n^2(\theta,\phi) * Sin\theta d\theta d\phi} = \frac{4\Pi}{\int_0^{\Pi}\int_0^{\Pi} Cos^4\theta * Sin^3\phi * Sin\theta d\theta d\phi}$$

$$= \frac{4\Pi}{\int_0^{\Pi} \sin\phi * (1 - Cos^2\phi) d\phi \int_0^{\Pi} Cos^4\theta * Sin\theta d\phi} = \frac{4\Pi}{(-\frac{Cos^5\theta}{5})_0^{\Pi} \int_0^{\Pi} (Sin\phi d\phi - Sin\phi Cos^2\phi d\phi)}$$

Dr. Gehan Sami
M.M. Elsherbini

$$= \frac{4\Pi}{(\frac{2}{5})[(-Cos\phi)_0^{\Pi} + (\frac{Cos^3\phi}{3})_0^{\Pi}]} = \frac{4\Pi}{(\frac{2}{5})(\frac{4}{3})} = 23.56$$

(b) To calculate D approximate.

We calculate θ_{max} ➔ ($Cos^2\theta_{max}$ =1..(max)) ➔ at $\theta_{max} = 0^o$,

We calculate Φ_{max} ➔ ($Sin^{3/2}\Phi_{max}$ =1..(max)) ➔ at $\Phi_{max} = 90^o$,

We calculate θ_h ➔ ($Cos^2\theta_h = \frac{1}{\sqrt{2}}$) ➔ $\theta_h = 33^o$

We calculate Φ_h ➔($Sin^{3/2}\Phi_h = \frac{1}{\sqrt{2}}$) ➔ $\Phi_{h=}52.5^o$

So: $\theta_{HP} = 2*|0-33^o| = 66^o$.

By the same way

We calculate $\Phi_{HP}=2*|90^o - 52.5^o|=75^o$

So: D approximate = $\frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{66*75} = 8.33.$

(c) decibel difference = 10log$\frac{23.5}{8.33} = 4.5$

9. In target-search ground-mapping radars it is desirable to have echo power received from a target, of constant cross section, to be independent of its range. For one such application, the desirable radiation intensity of the antenna is given by

$$U(\theta,\phi) = \left\{ \begin{array}{cc} 1 & 0^\circ \le \theta < 20^\circ \\ 0.342\csc(\theta) & 20^\circ \le \theta < 60^\circ \\ 0 & 60^\circ \le \theta \le 180^\circ \end{array} \right\} 0^\circ \le \phi \le 360^\circ$$

Find the directivity (in dB) using the exact formula.

Dr. Gehan Sami
M.M. Elsherbini

$$U(\theta,\phi)=\begin{cases} 1 & 0^\circ \le \theta \le 20^\circ \\ 0.342\csc(\theta) & 20^\circ \le \theta \le 60^\circ \\ 0 & 60^\circ \le \theta \le 180^\circ \end{cases} \quad 0^\circ \le \phi \le 360^\circ$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta \, d\phi = 2\pi\left[\int_0^{20^\circ}\sin\theta\, d\theta + \int_{20^\circ}^{60^\circ} 0.342\csc(\theta) \times \sin\theta \, d\theta\right] = 2\pi\left\{-\cos\theta\Big|_0^{\pi/9} + 0.342\cdot\theta\Big|_{\pi/9}^{\pi/3}\right\}$$

$$= 2\pi\left\{\left[-\cos\left(\frac{\pi}{9}\right)+1\right] + 0.342\left(\frac{\pi}{3}-\frac{\pi}{9}\right)\right\}$$

$$= 2\pi\left\{\left[-0.93969+1\right] + 0.342\pi\left(\frac{2}{9}\right)\right\}$$

$$= 2\pi\left\{0.06031 + 0.23876\right\} = 1.87912$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(1)}{1.87912} = 6.68737 = 8.25255 \text{ dB.}$$

10. The normalized radiation intensity of a given antenna is given by
(a) $U=\sin\theta\sin\varphi$, (b) $U=\sin\theta\sin^2\varphi$, (C) $U=\sin^2\theta\sin^3\varphi$
The intensity exists only in the $0\le\theta\le\pi$, $0\le\varphi\le\pi$ region, and it is zero elsewhere. Find the
(a) Exact directivity (dimensionless and in dB).
(b) Azimuthal and elevation plane half-power beam widths (in degrees).

Dr. Gehan Sami
M.M. Elsherbini

$$D_o = \frac{4\pi\, U_{max}}{P_{rad}}$$

(a) $U = \sin\theta \sin\phi$ for $0 \le \theta \le \pi$, $0 \le \phi \le \pi$

$U|_{max} = 1$ and it occurs when $\theta = \phi = \pi/2$.

$$P_{rad} = \int_0^\pi \int_0^\pi U \sin\theta \, d\theta \, d\phi = \int_0^\pi \sin\phi \, d\phi \int_0^\pi \sin^2\theta \, d\theta = 2\left(\frac{\pi}{2}\right) = \pi.$$

Thus $D_o = \frac{4\pi(1)}{\pi} = 4 = 6.02\,dB$

The half-power beamwidths are equal to

HPBW (az.) $= 2[90° - \sin^{-1}(1/2)] = 2(90° - 30°) = 120°$

HPBW (el.) $= 2[90° - \sin^{-1}(1/2)] = 2(90° - 30°) = 120°$

In a similar manner, it can be shown that for

(b) $U = \sin\theta \sin^2\phi \Rightarrow D_o = 5.09 = 7.07\,dB$

HPBW (el.) $= 120°$, HPBW (az.) $= 90°$

(C)

$U = \sin^2\theta \sin^3\phi \Rightarrow D_o = 9\pi/4 = 7.07 = 8.49\,dB$

HPBW (el.) $= 90°$, HPBW (az.) $= 74.93°$

11. Find the directivity (dimensionless and in dB) for the antenna of Problem 4 using Kraus' approximate formula.

(a)

$U = \sin\theta \cdot \sin\phi$; (a) $D_o \simeq \frac{41\,253}{\Theta_{1d}\,\Theta_{2d}} = \frac{41\,253}{120\,(120)} = 2.86 = 4.57\,dB$

(b) $D_o \simeq 3.82 = 5.82\,dB$

(c) $D_o \simeq 6.12 = 7.87\,dB$

12. The normalized radiation intensity of an antenna is rotationally symmetric in φ, and it is represented by

$$U = \begin{cases} 1 & 0^\circ \le \theta < 30^\circ \\ 0.5 & 30^\circ \le \theta < 60^\circ \\ 0.1 & 60^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases}$$

(a) What is the directivity (above isotropic) of the antenna (in dB)?

(a) $D_o = \dfrac{4\pi U_{max}}{P_{rad}} = \dfrac{U_{max}}{U_o}$

$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi} U \sin\theta \, d\theta = 2\pi \Big\{ \int_0^{30^\circ} \sin\theta \, d\theta +$

$\int_{30^\circ}^{60^\circ} (0.5)\sin\theta \, d\theta + \int_{60^\circ}^{90^\circ} (0.1)\sin\theta \, d\theta \Big\} = 2\pi \Big\{ (-\cos\theta)\Big|_0^{30^\circ} + \left(-\frac{\cos\theta}{2}\right)\Big|_{30^\circ}^{60^\circ} + (-0.1\cos\theta)\Big|_{60^\circ}^{90^\circ} \Big\}$

(Cont'd) $= 2\pi \left\{ (-0.866+1) + \left(\frac{-0.5+0.866}{2} \right) + \left(\frac{-0+0.5}{10} \right) \right\}$

$P_{rad} = 2\pi \{ -0.866 + 1 - 0.25 + 0.433 + 0.05 \} = 2\pi (0.367)$
$= 0.734 \cdot \pi = 2.3059$

$D_o = \dfrac{1(4\pi)}{2.3059} = 5.4496 = 7.3636 \text{ dB}$

13. The radiation intensity of an antenna is given by $\mathbf{U(\theta,\varphi)=\cos^4\theta \sin^2\varphi}$, for $0\le\theta\le\pi/2$ and $0\le\varphi\le2\pi$ (i.e., in the upper half-space). It is zero in the lower half-space.

Find the
(a) Exact directivity (dimensionless and in dB)
(b) Elevation plane half-power beam width (in degrees).

Dr. Gehan Sami
M.M. Elsherbini

(a) $P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\sin^2\phi\, d\phi \cdot \int_0^{\pi/2}\cos^4\theta \sin\theta\, d\theta$

$= (\pi)\left(\frac{1}{5}\right) = \frac{\pi}{5}.$

$U_{max} = U(\theta = 0°, \phi = \pi/2) = 1.$

$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi}{(\pi/5)} = 20 = 13.0 \text{ dB}$

(b) Elevation Plane : θ varies , ϕ fixed

→ Choose $\phi = \pi/2$.

$U(\theta, \phi = \pi/2) = \cos^4\theta \quad , \quad 0 \leq \theta \leq \pi/2.$

$\cos^4\left[\frac{HPBW(el.)}{2}\right] = \frac{1}{2}$

$HPBW(el.) = 2 \cdot \cos^{-1}\{\sqrt{0.5}\} = 65.5°$

14. The far-zone electric-field intensity (array factor) of an end-fire two-element array antenna, placed along the z-axis and radiating into free-space, is given by

$$E = \cos\left[\frac{\pi}{4}(\cos\theta - 1)\right]\frac{e^{-jkr}}{r}, \qquad 0 \leq \theta \leq \pi$$

Find the directivity using Kraus' approximate formula

(a). $E\big|_{max} = \cos\left[\frac{\pi}{4}(\cos\theta - 1)\right]\Big|_{max} = 1 \quad$ at $\theta = 0°$.

$0.707\, E_{max} = 0.707 \cdot (1) = \cos\left[\frac{\pi}{4}(\cos\theta_1 - 1)\right]$

$\frac{\pi}{4}(\cos\theta_1 - 1) = \pm\frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(2) = \text{does not exist} \\ \cos^{-1}(0) = 90° = \frac{\pi}{2} \text{ rad.} \end{cases}$

$\Theta_{1r} = \Theta_{2r} = 2\left(\frac{\pi}{2}\right) = \pi$

$D_o \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.049 \text{ dB}$

15. The normalized far-zone field pattern of an antenna is given by

$$E = \begin{cases} (\sin\theta\cos^2\phi)^{1/2} & 0 \leq \theta \leq \pi \text{ and } 0 \leq \phi \leq \pi/2,\ 3\pi/2 \leq \phi \leq 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Dr. Gehan Sami

M.M. Elsherbini

Find the directivity using
(a) The exact expression
(b) Kraus' approximate formula

$$U = \frac{1}{2\eta}|E|^2 = \frac{1}{2\eta}\sin\theta\cos^2\phi \Rightarrow U_{max} = \frac{1}{2\eta}$$

(a). $P_{rad} = 2\cdot\int_0^{\pi/2}\int_0^{\pi}\frac{1}{2\eta}\sin^2\theta\cos^2\phi\, d\theta\, d\phi = \frac{1}{\eta}\left(\frac{\pi}{4}\right)\left(\frac{\pi}{2}\right) = \frac{\pi^2}{8\eta}$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi\left(\frac{1}{2\eta}\right)}{\frac{\pi^2}{8\eta}} = \frac{16}{\pi} = 5.09 = 7.07\,dB$$

(b). $U_{max} = \frac{1}{2\eta}$ at $\theta = \pi/2,\ \phi = 0$

In the elevation plane through the maximum $\phi = 0$ and $U = \frac{1}{2\eta}\sin\theta$.
The 3-dB point occurs when
$U = 0.5\,U_{max} = 0.5\left(\frac{1}{2\eta}\right) = \frac{1}{2\eta}\sin\theta_1 \Rightarrow \theta_1 = \sin^{-1}(0.5) = 30°$
Therefore $\Theta_{1d} = 2(90-30) = 120°$

In the azimuth plane through the maximum $\theta = \pi/2$ and $U = \frac{1}{2\eta}\cos^2\phi$.
The 3-dB point occurs when $U = 0.5\,U_{max} = 0.5\left(\frac{1}{2\eta}\right) = \frac{1}{2\eta}\cos^2\theta_1 \Rightarrow$
$\phi_1 = \cos^{-1}(0.707) = 45°$, $\Theta_{2d} = 2(90° - 45°) = 90°$

Therefore using Kraus' formula $D_0 \simeq \frac{41,253}{120\cdot(90)} = 3.82 = 5.82\,dB$

16. Estimate the directivity for a source with relative field pattern

a. $E = \cos 2\theta \cos\theta$. Assume a unidirectional pattern.
b. $E = \sin\left(\frac{\Pi}{2}\cos\theta\right)$. Assume $0 \le \theta \le \Pi$ & $0 \le \varphi \le 2\Pi$.

(a) ***Dexact*** $= \dfrac{4\Pi}{\int_0^{\Pi}\int_0^{\Pi}(\cos 2\theta\cos\theta)^2 * \sin\theta\, d\theta\, d\phi}$

$$= \frac{4\Pi}{\Pi * \int_0^{\Pi}(\cos 2\theta\cos\theta)^2 * \sin\theta\, d\theta}$$

$$= \frac{4}{\int_0^{\Pi}(2Cos^2\theta-1)^2 Cos^2\theta * Sin\theta d\theta} = \frac{4}{\int_0^{\Pi}(4Cos^4\theta-4Cos^2\theta+1)Cos^2\theta * Sin\theta d\theta}$$

$$= \frac{4}{\int_0^{\Pi}(4Cos^6\theta-4Cos^4\theta+Cos^2\theta) * Sin\theta d\theta} =$$

$$\frac{4}{-(\frac{4Cos^7\theta}{7}-4\frac{Cos^5\theta}{5}+\frac{Cos^3\theta}{3})_0^{\Pi}} = 19.1$$

(b)Dexact$$= \frac{4\Pi}{\int_0^{\Pi}\int_0^{\Pi}(Sin^2(\frac{\Pi}{2}Cos\theta) * Sin\theta d\theta d\phi} =$$

$$\frac{4\Pi}{2\Pi\int_0^{\Pi}(Sin^2(\frac{\Pi}{2}Cos\theta) * Sin\theta d\theta d\phi}$$

$$= \frac{2}{\int_0^{\Pi}(Sin^2(\frac{\Pi}{2}Cos\theta) * Sin\theta d\theta d\phi}$$

Let $(\frac{\Pi}{2}Cos\theta) = x$ *→ (1)*

So $\frac{dx}{d\theta} = -\frac{\Pi}{2}Sin\theta$ ➔ $Sin\theta d\theta = \frac{-2}{\Pi}dx$ *→ (2)*

Replace (θ from 0 to Π) to (x from $\frac{\Pi}{2}$ *to* $\frac{-\Pi}{2}$*). By substituting in (1).*

Dexact$$= \frac{2}{\int_{\frac{\Pi}{2}}^{\frac{-\Pi}{2}}(Sin^2(x) * \frac{-2}{\Pi}dx} = \frac{-\Pi}{\int_{\frac{\Pi}{2}}^{\frac{-\Pi}{2}}(Sin^2(x) * dx} = \frac{-\Pi}{\int_{\frac{\Pi}{2}}^{\frac{-\Pi}{2}}\frac{1}{2}(1-Cos2x)dx} =$$

Dr. Gehan Sami
M.M. Elsherbini

$$\frac{-\Pi}{\frac{1}{2}\left(x-\frac{Sin2x}{2}\right)_{\frac{\Pi}{2}}^{\frac{-\Pi}{2}}}=2.$$

(REPORT)

1. The normalized radiation intensity of an antenna is symmetric, and it can be approximated by

$$U(\theta)=\begin{cases}1 & 0^\circ \le \theta < 30^\circ \\ \dfrac{\cos(\theta)}{0.866} & 30^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ\end{cases}$$

And it is independent of φ. Find the
(a) Exact directivity by integrating the function
(b) Approximate directivity using Kraus' formula.

2. (a) $$P_{rad}=\int_0^{2\pi}\int_0^{\pi}U(\theta,\phi)\sin\theta\, d\theta\, d\phi = 2\pi\cdot\left\{\int_0^{30^\circ}\sin\theta\, d\theta+\int_{30^\circ}^{90^\circ}\frac{\cos\theta\sin\theta}{0.866}d\theta\right\}$$

$$=2\pi\left\{\int_0^{\pi/6}\sin\theta\, d\theta+\int_{\pi/6}^{\pi/2}\frac{1}{0.866}\cos\theta\cdot\sin\theta\, d\theta\right\}$$

$$=2\pi\left\{-\cos\theta\Big|_0^{\pi/6}+\frac{1}{0.866}\left(-\frac{\cos^2\theta}{2}\right)\Big|_{\pi/6}^{\pi/2}\right\}=2\pi\left[-0.866+1+0.433\right]=3.5626$$

$$D_0=\frac{4\pi U_{max}}{P_{rad}}=\frac{4\pi(1)}{3.5626}=3.5273=5.4745\text{ dB}$$

(b) $$U=\frac{\cos(\theta)}{0.866}=0.5\Rightarrow\cos\theta=0.5(0.866)=0.433,\quad \theta=\cos^{-1}(0.433)=64.34^\circ$$

$$\Theta_{1r}=2(64.34)=128.68^\circ=2.246\text{ rad}=\Theta_{2r}$$

$$D_0\simeq\frac{4\pi}{\Theta_{1r}\Theta_{2r}}=\frac{4\pi}{(2.246)^2}=2.4912=3.9641\text{ dB}$$

2. Repeat Problem 8 when

$$E=\cos\left[\frac{\pi}{4}(\cos\theta+1)\right]\frac{e^{-jkr}}{r},\qquad 0\le\theta\le\pi$$

Dr. Gehan Sami
M.M. Elsherbini

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a. $E|_{max} = \cos\left(\frac{\pi}{4}(\cos\theta+1)\right)\Big|_{max} = 1$ at $\theta = \pi$.

$0.707 = \cos\left(\frac{\pi}{4}(\cos\theta_1+1)\right)$

$\frac{\pi}{4}(\cos\theta_1+1) = \pm\frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(-2) \to \text{does not exist.} \\ \cos^{-1}(0) \to 90^\circ \to \frac{\pi}{2}\ rad \end{cases}$

$\theta_{1r} = \theta_{2r} = 2\left(\frac{\pi}{2}\right) = \pi$.

$D_0 \simeq \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.048\ dB$

3. The normalized radiation intensity of an antenna is represented by

$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \quad (0 \le \theta \le 90^\circ, \quad 0^\circ \le \phi \le 360^\circ)$

Find the exact and approximate directivity.

a. Since the $U(\theta)$ represents the *power* pattern, to find the half-power beamwidth you set the function equal to half of its maximum, or

$$U(\theta)|_{\theta=\theta_h} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_h} = 0.5 \Rightarrow \cos\theta_h \cos 3\theta_h = 0.707$$

$$\theta_h = \cos^{-1}\left(\frac{0.707}{\cos 3\theta_h}\right)$$

Since this is an equation with transcendental functions, it can be solved iteratively. After a few iterations, it is found that

$$\theta_h \approx 0.25\ radians = 14.325^\circ$$

Since the function $U(\theta)$ is symmetrical about the maximum at $\theta = 0$, then the HPBW is

$$\text{HPBW} = 2\theta_h \approx 0.50\ radians = 28.65^\circ$$

ونكمل بقانون الدايركتيفيتي

Dr. Gehan Sami
M.M. Elsherbini

Good Luck

Dr. Gehan Sami
M.M. Elsherbini